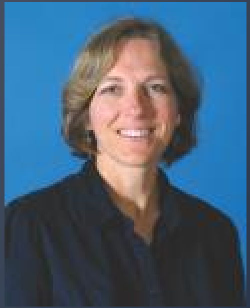
Georgia Institute of Technology

Introduction to Electronics

An introduction to electronic components and a study of circuits containing such devices.

School of Electrical and Computer Engineering

Week 3:
Op Amps Part 2



First-Order Lowpass Filters

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Computer Engineering

Introduce lowpass filters

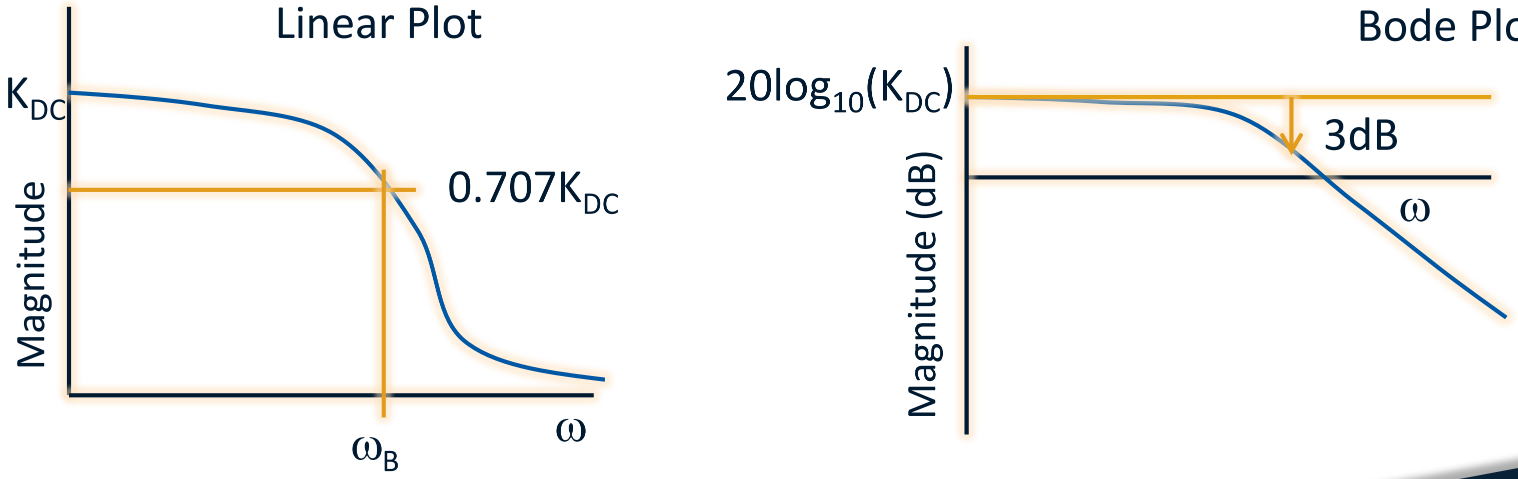
Lesson Objectives

- Introduce active lowpass filters

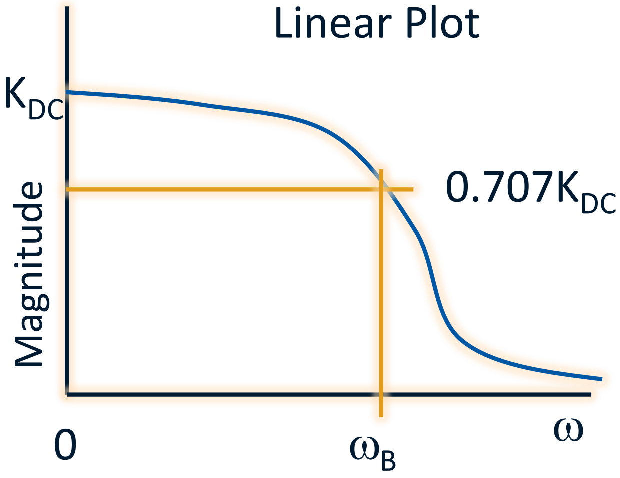
Lowpass Filters

- Lowpass filters pass low frequency components and attenuate high frequency components

Transfer Function H(ω)



First-Order Filter

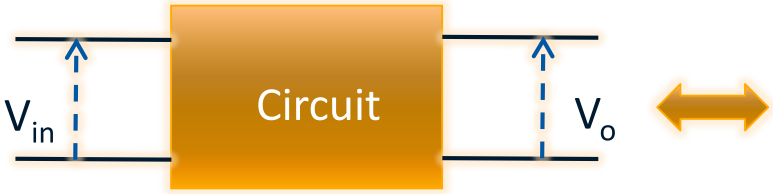


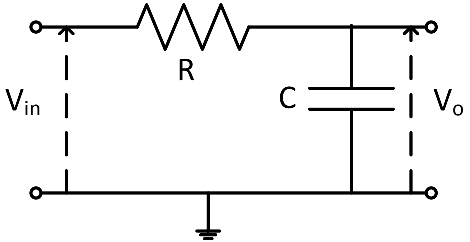
$$H(\omega) = K_{DC}\frac{1}{\tau j\omega + 1}$$

Bandwidth, $\omega_B = 1/\tau$

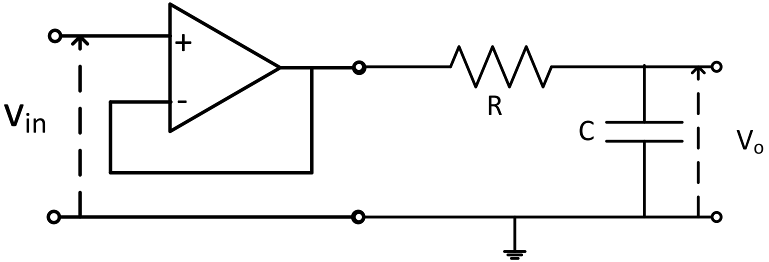
DC Gain = H(0) = K_{DC}

From Passive to Active Lowpass Filters

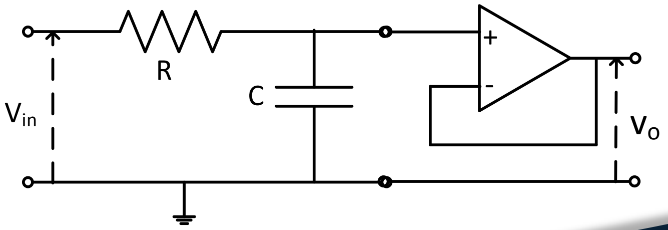




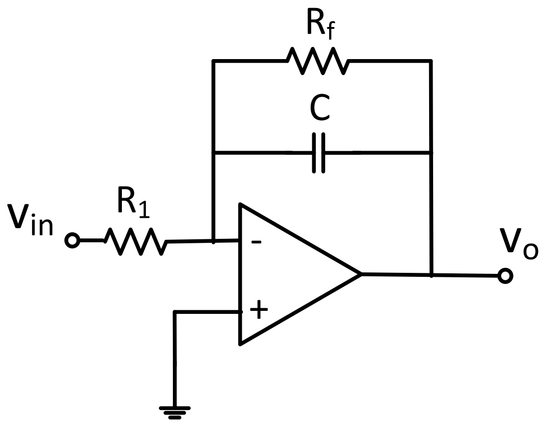
Isolation at the input:



Isolation in the output:



First-Order Inverting Lowpass Filter



$$V_o = -\frac{R_f}{R_1}\frac{1}{R_f C j\omega + 1}V_{in}$$

Frequency Characteristics of LP Filter

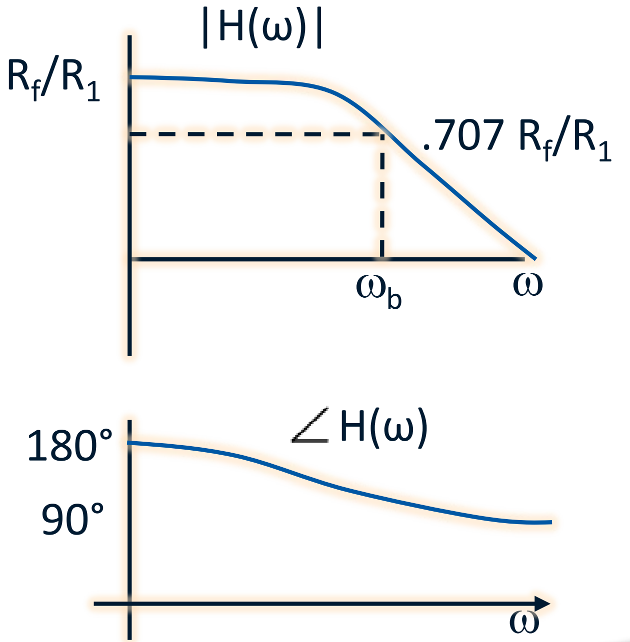
$$H(\omega) = -\frac{R_f}{R_1}\frac{1}{(R_f Cj\omega + 1)}$$

$$|H(\omega)| = \frac{R_f}{R_1}\frac{1}{\sqrt{(R_f C_f \omega)^2 + 1}}$$

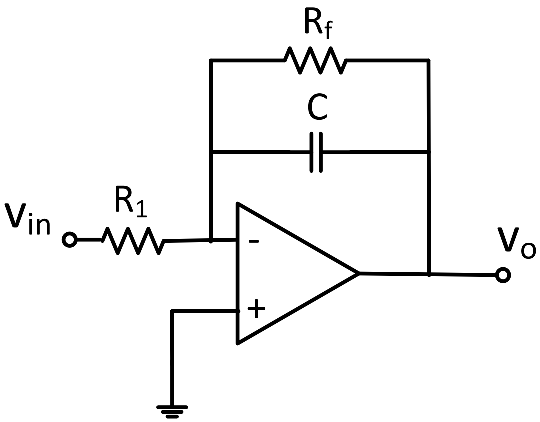
$$\angle H(\omega) = 180 - \arctan(R_f C_f \omega)$$

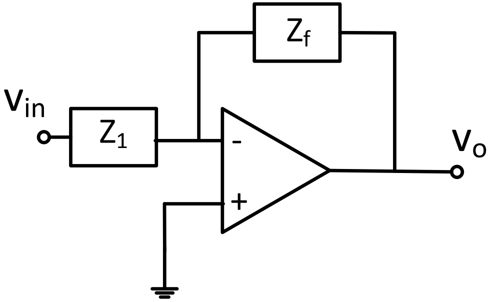
$$\text{DC Gain} = -\frac{R_f}{R_1}$$

$$\text{Bandwidth}, \omega_b = \frac{1}{R_f C_f}$$



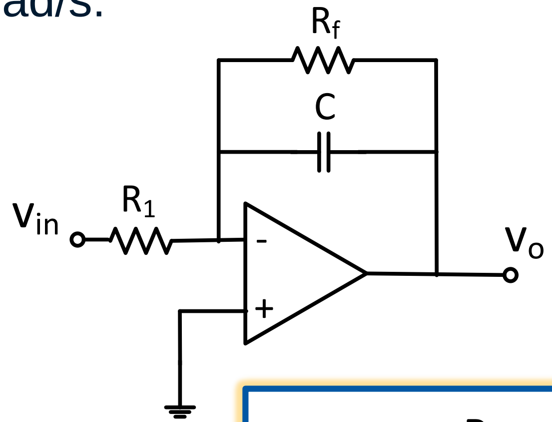
Derivation: Lowpass Filter





Example

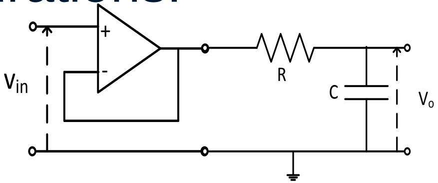
Design an inverting lowpass filter to have a DC gain of -2 and a bandwidth of 500 rad/s:



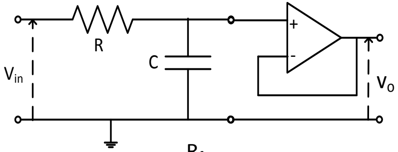
$$\boxed{H(\omega) = -\frac{R_f}{R_1}\frac{1}{R_f C j\omega + 1}}$$

Summary

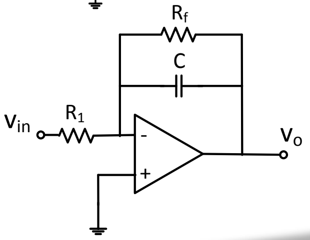
- A **lowpass filter** passes low frequency signals and attenuates high frequency signals
- Three first-order lowpass configurations:
  - Noninverting, isolation at the input

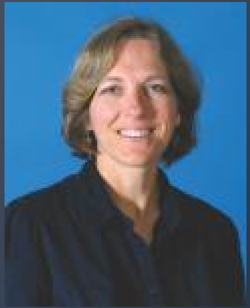


  - Noninverting, isolation at the output



  - Inverting, isolation at input and output





First-Order Highpass Filters

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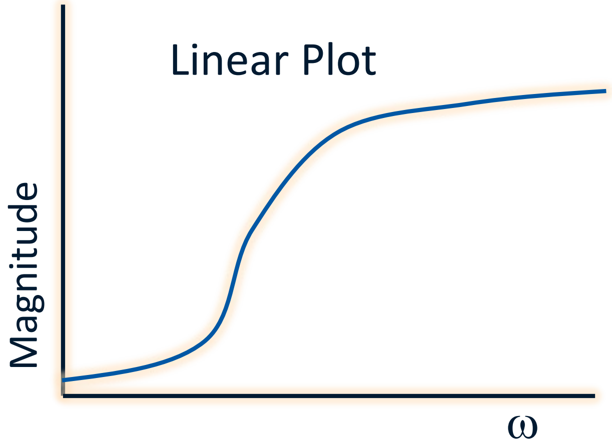
Introduce highpass filters

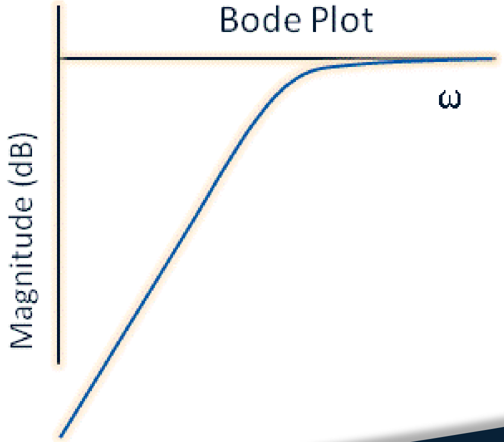
Lesson Objectives

- Introduce active highpass filters

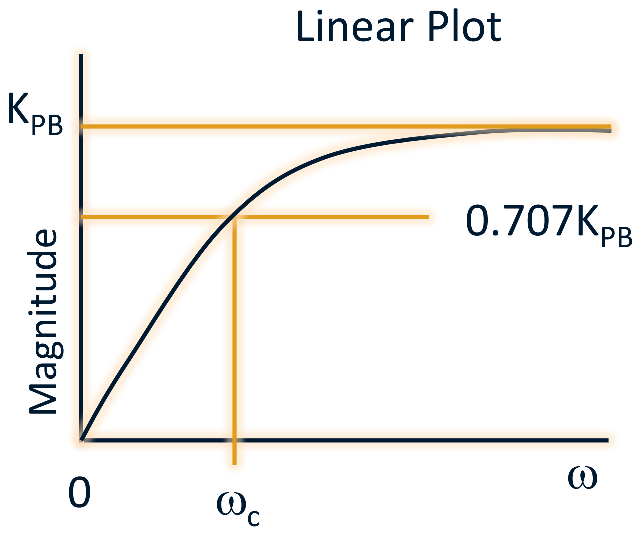
Highpass Filter

- Passes high frequency components and attenuates low frequency components





First-Order Filter

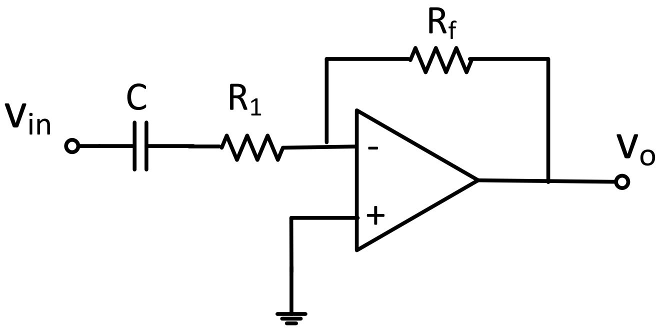


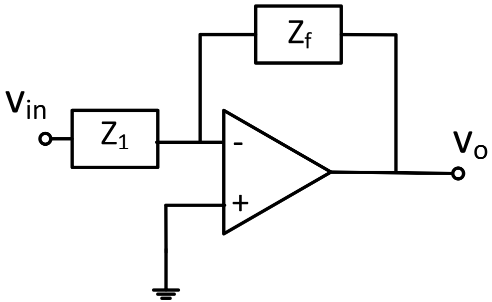
$$H(\omega) = \frac{Kj\omega}{\tau j\omega + 1}$$

Corner Frequency, $\omega_c = 1/\tau$

Passband Gain= $K_{PB} = K/\tau$

Inverting Highpass Filter Configuration





$$\boxed{V_o = \frac{-R_f C j\omega}{(R_1 C j\omega + 1)} V_{in}}$$

Frequency Characteristics of HP Filter

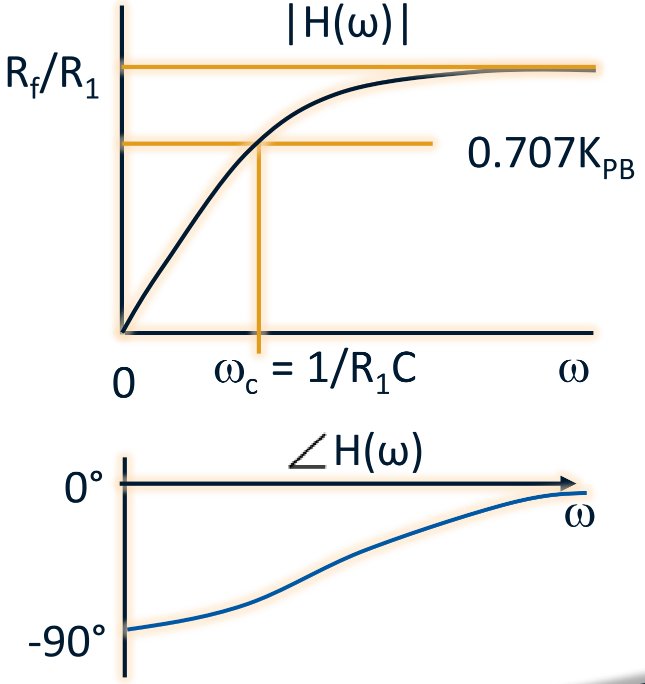
$$H(\omega) = \frac{-R_f C j\omega}{(R_1 C j\omega + 1)}$$

$$|H(\omega)| = \frac{R_f C\omega}{\sqrt{(R_1 C\omega)^2 + 1}}$$

$$\angle H(\omega) = -90^\circ - \arctan(R_1 C\omega)$$

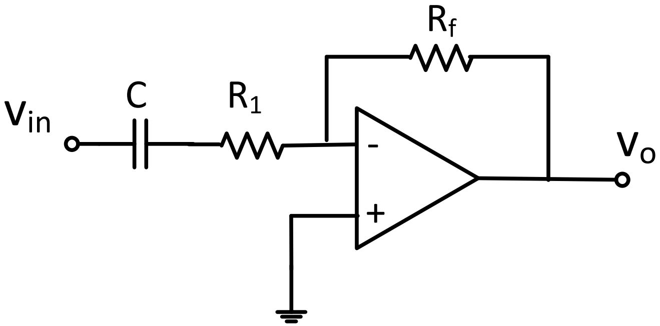
$$\text{Passband Gain}\,(\omega \to \infty) = -\frac{R_f}{R_1}$$

$$\text{Corner Freq., } \omega_c = \frac{1}{R_1 C}$$



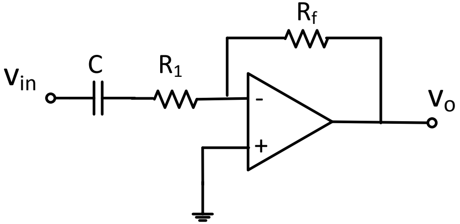
Example

Design a highpass filter to have a passband gain of 2 and a corner frequency of 1k rad/s:



Summary

- A **highpass filter** passes high frequency components in signals and attenuates low frequency components
- First-order highpass filter



$$H(\omega) = \frac{-R_f C j\omega}{(R_1 C j\omega + 1)}$$

- Design based on
  - Corner frequency of the passband, ω_c
  - Passband gain, K_{PB}

Introduction to Electronics

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Cascaded First-Order Filters

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Introduce cascaded first-order op-amp filters

Previous Lesson

- Introduced op-amp first-order highpass filters

Lesson Objectives

- Introduce cascaded filters
- Introduce bandpass filter characteristics

Transfer Functions in Hertz f

Lowpass

$$H(\omega) = K_{DC} \frac{1}{\tau j\omega + 1}$$

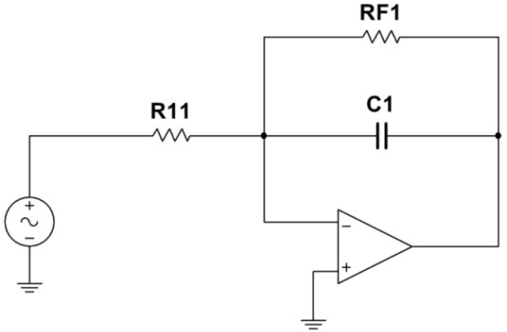
Bandwidth, $\omega_B = 1/\tau$
DC Gain = H(0) = K_{DC}

Highpass

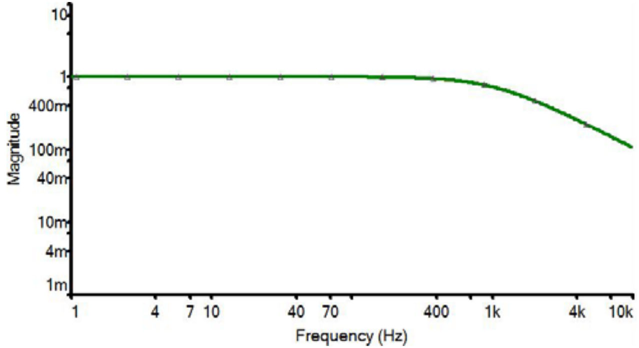
$$H(\omega) = \frac{Kj\omega}{\tau j\omega + 1}$$

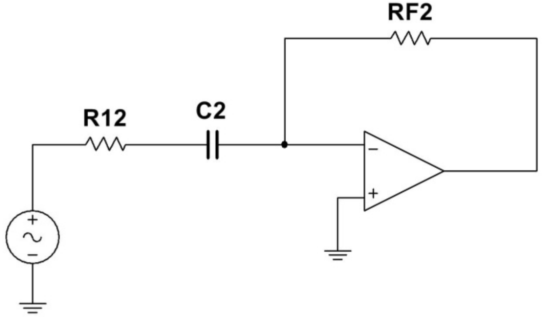
Corner Frequency, $\omega_c = 1/\tau$
Passband Gain= $K_{PB} = K/\tau$

First-Order LPF and HPF

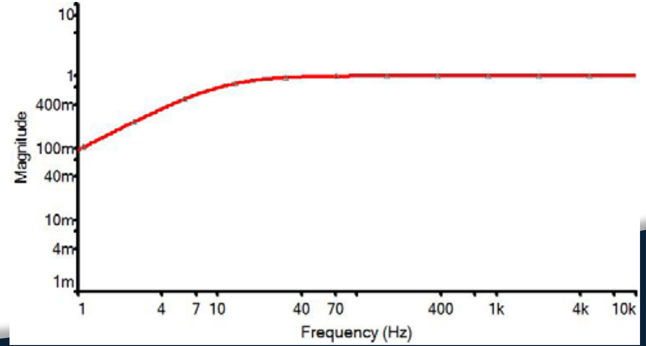


$$H_{LP}(f) = K_{DC}\frac{1}{\frac{jf}{f_0}+1}$$

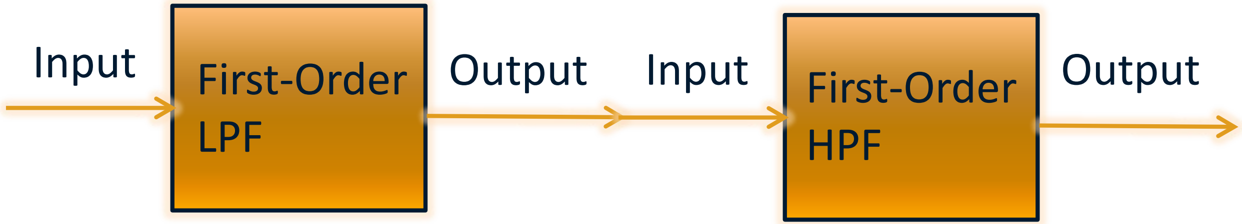




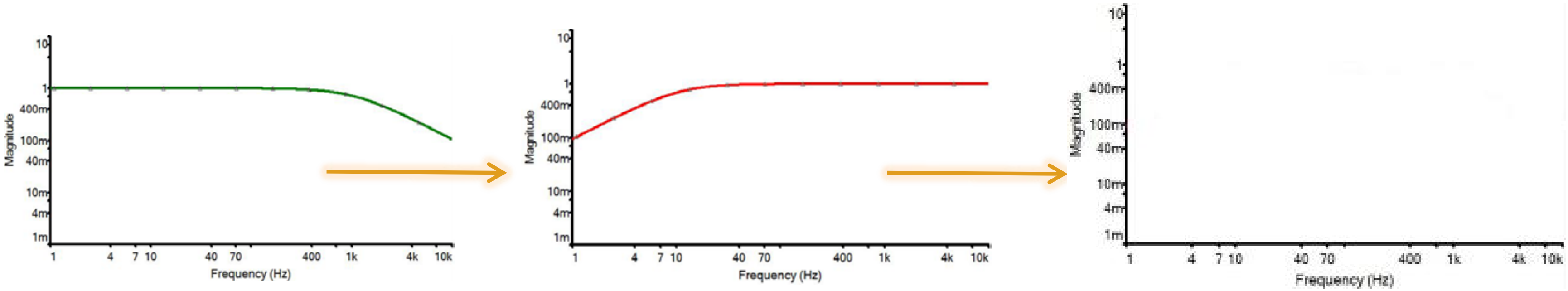
$$H_{HP}(f) = K_{PB}\frac{\frac{jf}{f_0}}{\frac{jf}{f_0}+1}$$



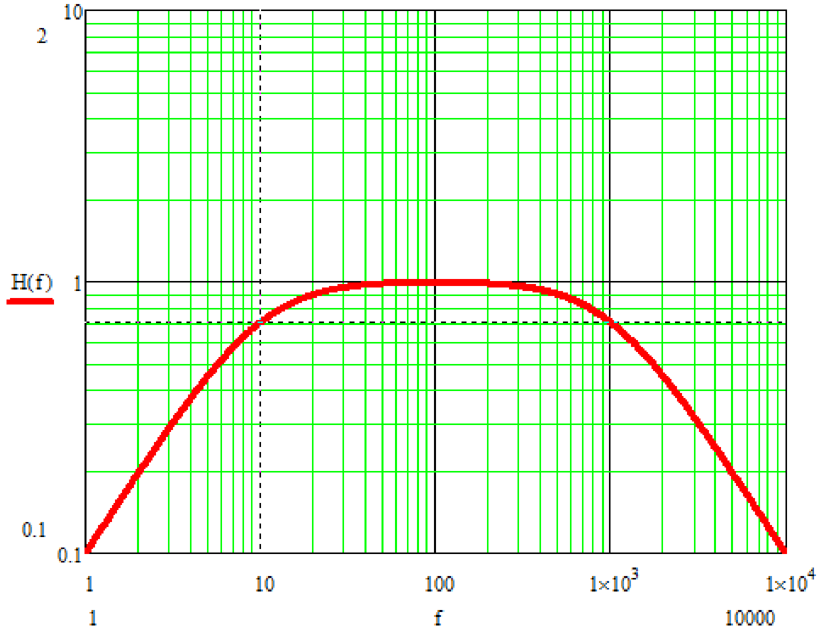
Cascaded Filter



Cascaded Filter



Bandpass Filter Characteristics



Cascaded Filter Transfer Function

$$H_{BP}(f) = H_{LP}(f)H_{HP}(f) = K_{DC}\frac{1}{\frac{jf}{f_{lp}}+1}K_{PB}\frac{\frac{jf}{f_{hp}}}{\frac{jf}{f_{hp}}+1}$$

$$K = K_{DC}K_{PB}\left(\frac{f_{lp}}{f_{lp}+f_{hp}}\right)$$

$$f_0 = \sqrt{f_{lp}f_{hp}}$$

$$Q = \frac{\sqrt{f_{lp}f_{hp}}}{f_{lp}+f_{hp}}$$

$$BW = f_{lp}+f_{hp}$$

Summary

- Cascaded Lowpass and Highpass Filters
- Bandpass Filter Characteristics

Next Lesson

- Second-Order Transfer Functions

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Second-Order Transfer Functions

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Introduce second-order filter transfer functions

Previous Lesson

- Introduced cascaded first-order op-amp filters

Lesson Objectives

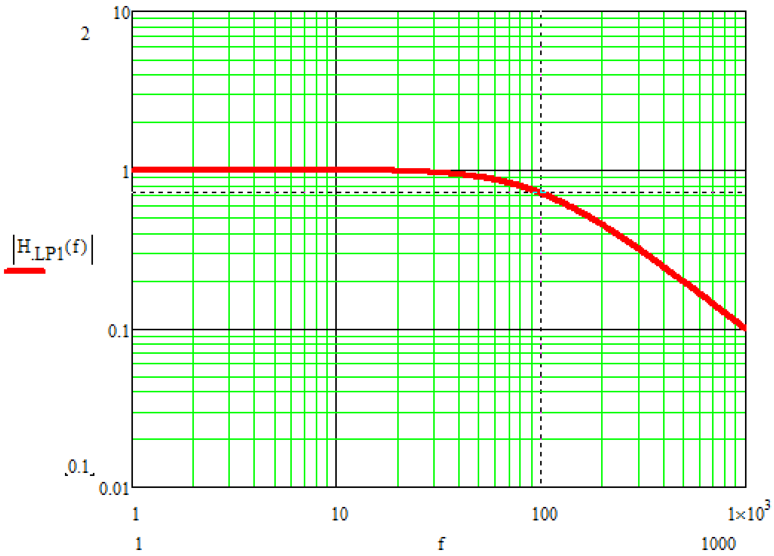
- Introduce second-order filter transfer functions
- Examine features of transfer functions

Filter Transfer Function

- Ratio of output voltage to input voltage as a function of frequency
- For any frequency, the transfer function is a complex number that indicates how the filter modifies the magnitude and phase of the input to produce the output

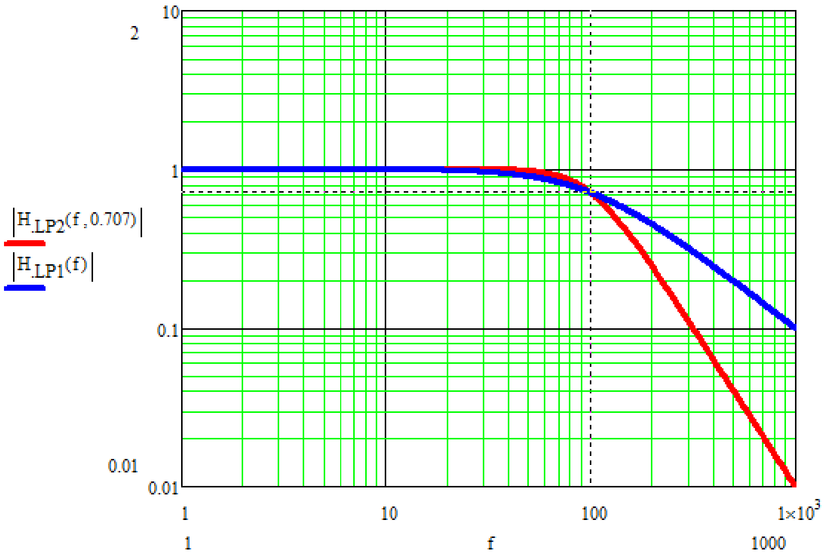
$$H(f) = \frac{V_{out}(f)}{V_{in}(f)}$$

First-Order Low-Pass Filter



$$H_{LP1}(f) = K\frac{1}{\frac{jf}{f_0}+1}$$

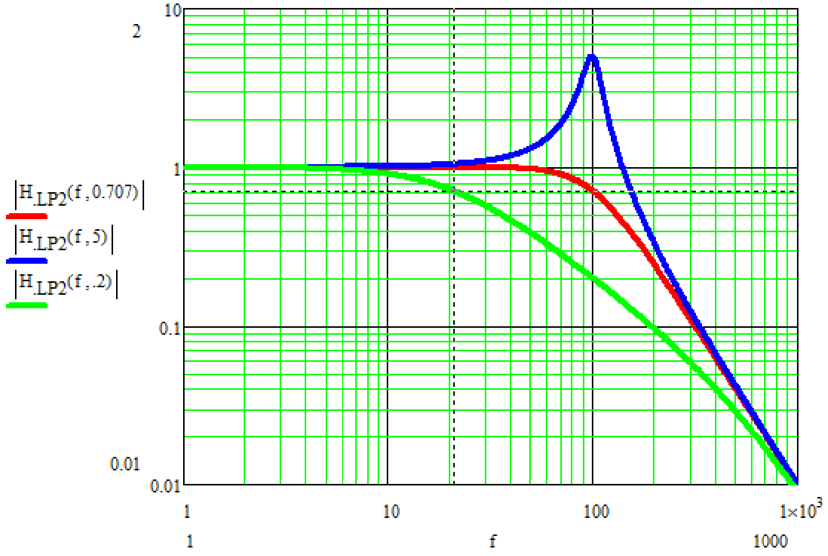
Second-Order Low-Pass Filter



$$H_{LP1}(f) = K\frac{1}{\frac{jf}{f_0}+1}$$

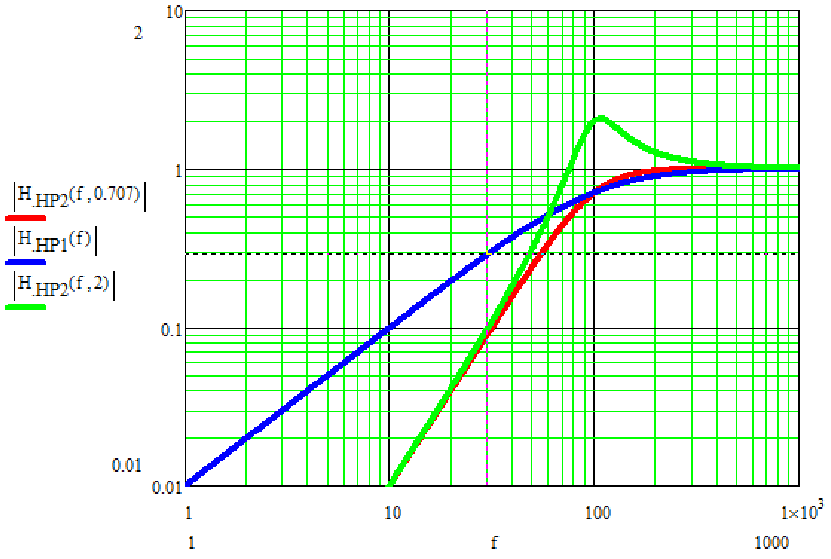
$$H_{LP2}(f) = K\frac{1}{\left(\frac{jf}{f_0}\right)^2+\frac{jf}{f_0}\frac{1}{Q}+1}$$

Effect of Quality Factor (Q)



$$H_{LP2}(f) = K\frac{1}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0}\frac{1}{Q} + 1}$$

High-Pass Filters

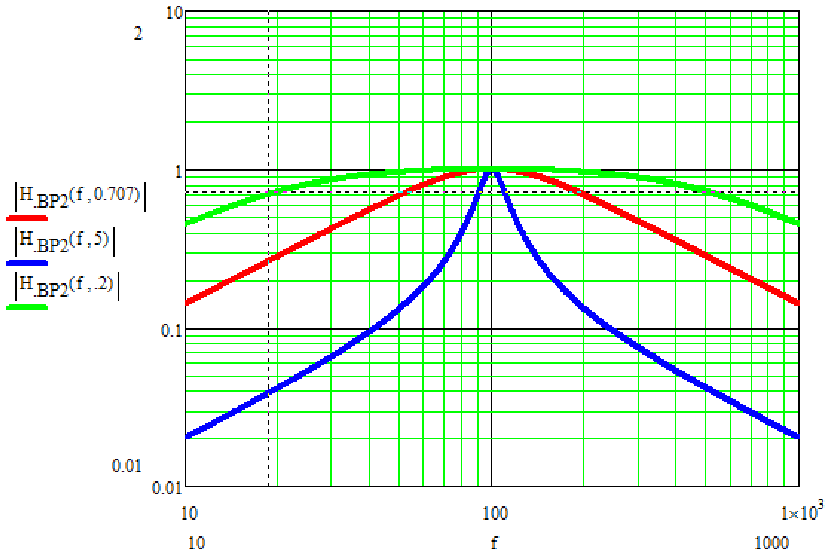


$$H_{HP1}(f) = K\frac{\frac{jf}{f_0}}{\frac{jf}{f_0}+1} \qquad H_{HP2}(f) = K\frac{\left(\frac{jf}{f_0}\right)^2}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0}\frac{1}{Q} + 1}$$

$$H_{LP1}(f) = K\frac{1}{\frac{jf}{f_0}+1} \qquad H_{LP2}(f) = K\frac{1}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0}\frac{1}{Q} + 1}$$

Band-Pass Filters

$$H_{BP2}(f) = K\frac{\frac{if}{f_0}\frac{1}{Q}}{\left(\frac{if}{f_0}\right)^2 + \frac{if}{f_0}\frac{1}{Q} + 1}$$



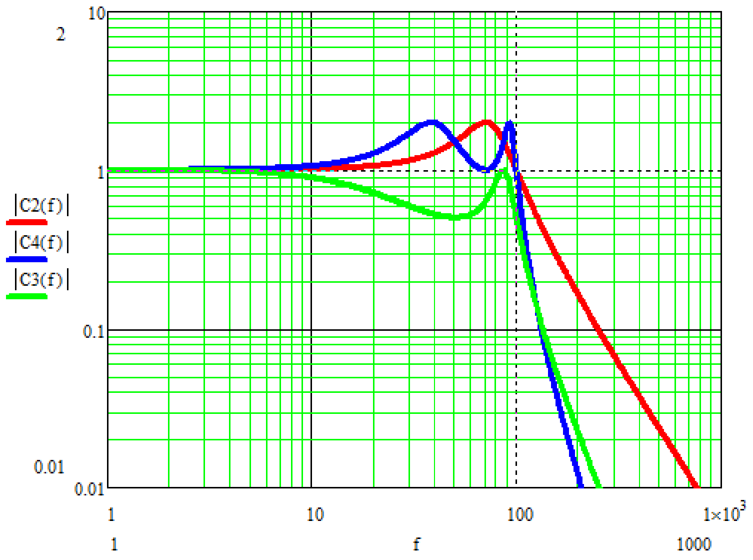
$$Q = \frac{f_0}{BW}$$

$$BW = Bandwidth = f_u - f_l$$

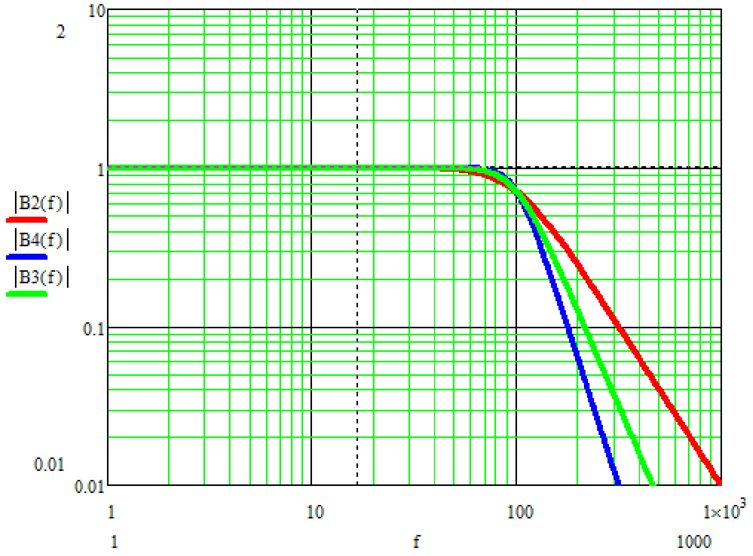
Butterworth and Chebyshev

- Types of transfer functions
- For second-order filters, the type is determined by the Q value
- $Q = 1/\sqrt{2}$ Butterworth (Maximally Flat)
- $Q > 1/\sqrt{2}$ Chebyshev

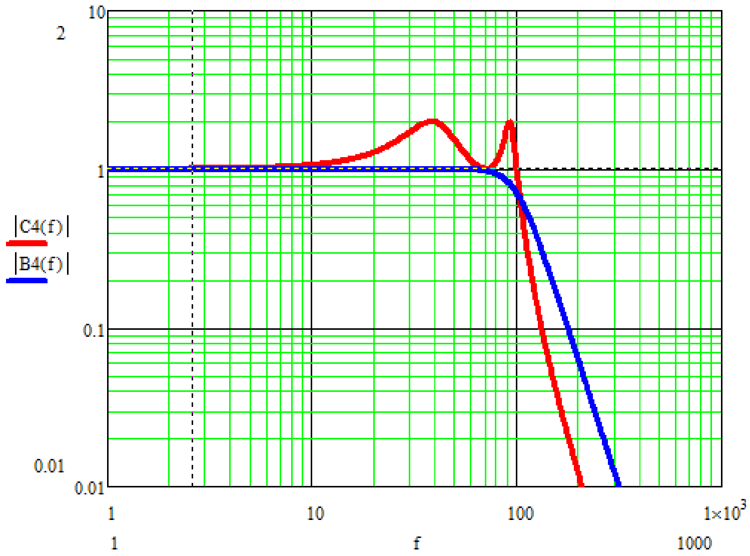
Chebyshev Filters



Butterworth Filters



Fourth-Order Butterworth vs. Chebyshev



Summary

- Introduced second-order transfer functions
- Examined features of transfer functions

Next Lesson

- Op-Amp Second-Order Filter Circuits

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Second-Order Filter Circuits

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Introduce second-order Sallen-Key filter circuits

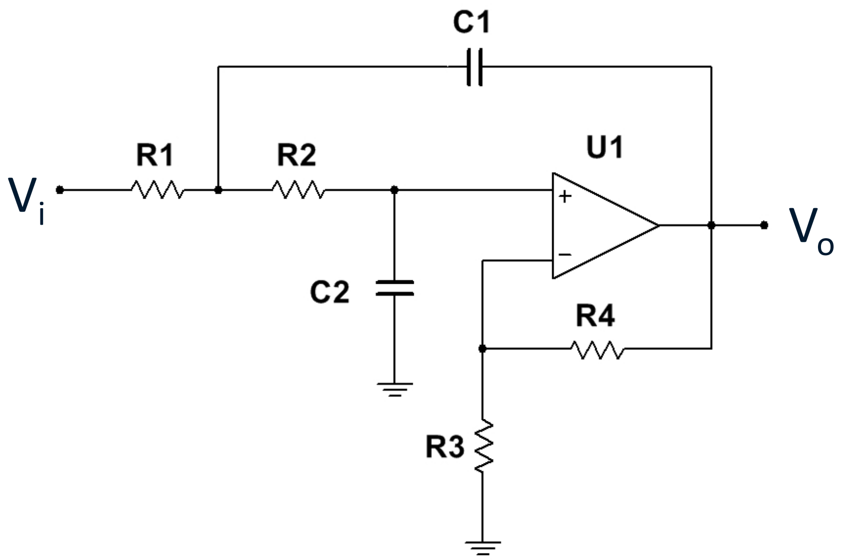
Previous Lesson

- Introduced second-order transfer functions

Lesson Objectives

- Introduce second-order filter circuits
- Design second-order filters

Sallen-Key Low-Pass Filter



$$H_{LP2}(f) = K \frac{1}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0}\frac{1}{Q} + 1}$$

$$K = 1 + \frac{R_4}{R_3}$$

$$f_o = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(1-K)R_1 C_1 + (R_1 + R_2)C_2}$$

Lowpass Design Equations

Special Case 1
(K = 1, Solve for C's)

$$K = 1 \quad (R_3 = \infty, R_4 = 0)$$

$$C_1 = \frac{Q}{\omega_o}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$C_2 = \frac{1}{Q\omega_o\left(R_1 + R_2\right)}$$

Can simplify with $R_1 = R_2$

Special Case 2
(K = 1, Solve for R's)

$$K = 1 \quad (R_3 = \infty, R_4 = 0)$$

$$R_1, R_2 = \frac{1}{2Q\omega_0 C_2}\left(1 \pm \sqrt{1 - 4Q^2\frac{C_2}{C_1}}\right)$$

$$4Q^2 C_2/C_1 \leq 1$$

R_1 and R_2 are interchangeable

Special Case 3
(R's equal and C's equal)

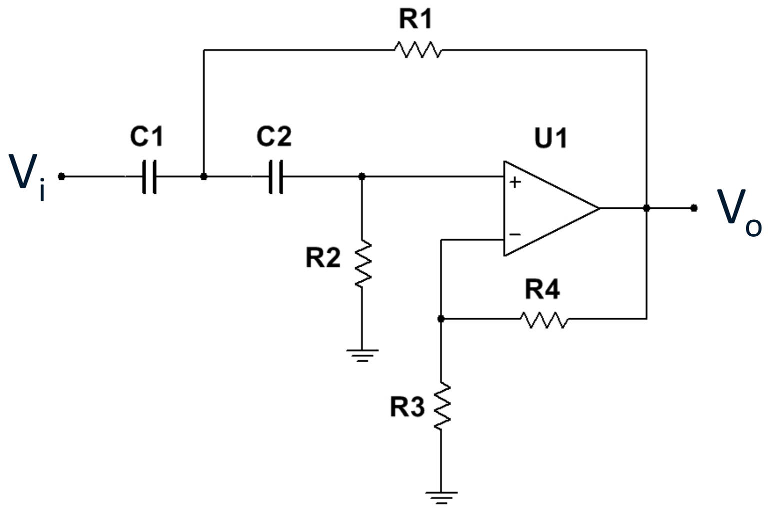
$$R_1 = R_2 = R \qquad K = 1 + \frac{R_4}{R_3}$$

$$C_1 = C_2 = C$$

$$K = 3 - \frac{1}{Q}$$

$$R = \frac{1}{\omega_0 C}$$

Sallen-Key Highpass Filter



$$H_{HP2}(f) = K \frac{\left(\frac{jf}{f_0}\right)^2}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0}\frac{1}{Q} + 1}$$

$$K = 1 + \frac{R_4}{R_3}$$

$$f_o = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(1-K)R_2 C_2 + (C_1 + C_2)R_1}$$

Highpass Design Equations

Special Case 1
($K = 1$, $C_1 = C_2 = C$)

$$K = 1 \quad (R_3 = \infty, R_4 = 0)$$

$$C_1 = C_2 = C$$

$$R_1 = \frac{1}{2Q\omega_o C}$$

$$R_2 = \frac{2Q}{\omega_0 C}$$

Special Case 2
(R's equal and C's equal)

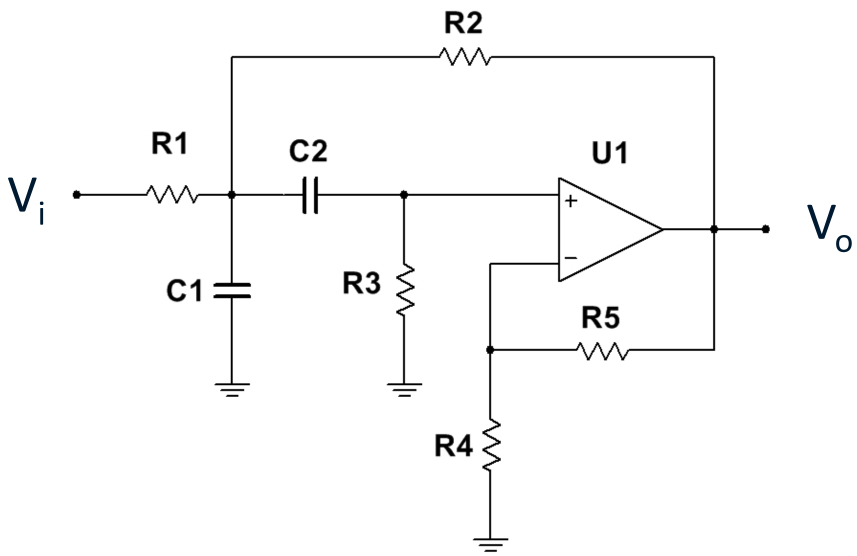
$$R_1 = R_2 = R \qquad K = 1 + \frac{R_4}{R_3}$$

$$C_1 = C_2 = C$$

$$K = 3 - \frac{1}{Q}$$

$$C = \frac{1}{\omega_0 R}$$

Sallen-Key Bandpass Filter



$$H_{BP2}(f) = K \frac{\frac{jf}{f_0}\frac{1}{Q}}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0}\frac{1}{Q} + 1}$$

$$K = \frac{R_2}{R_1 + R_2} \frac{K_0 R_3 C_2}{(R_1 \| R_2)(C_1 + C_2) + R_3 C_2 \left[1 - \frac{K_0 R_1}{(R_1+R_2)}\right]}$$

$$f_0 = \frac{1}{2\pi\sqrt{(R_1 \| R_2) R_3 C_1 C_2}}$$

$$Q = \frac{\sqrt{(R_1 \| R_2) R_3 C_1 C_2}}{(R_1 \| R_2)(C_1 + C_2) + R_3 C_2 \left[1 - \frac{K_0 R_1}{(R_1+R_2)}\right]}$$

$$K_0 = 1 + \frac{R_5}{R_4}$$

Bandpass Design Equations

Special Case
(R's equal and C's equal)

$$R_1 = R_2 = R_3 = R \qquad K_0 = 1 + \frac{R_5}{R_4}$$

$$C_1 = C_2 = C$$

$$R = \frac{\sqrt{2}}{2\pi f_0 C}$$

$$K_0 = 4 - \frac{1}{Q^2}$$

$$K = 4Q^2 - 1$$

Notch Filters

$$H_{BR2}(f) = K \frac{\left(\frac{jf}{f_0}\right)^2 + 1}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0}\frac{1}{Q} + 1}$$

Summary

- Introduced second-order filter circuits

Next Lesson

- Filter Design Example

Introduction to Electronics

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An introduction to electronic components and a study of circuits containing such devices.

Lowpass Filter Design Example

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Design a second-order Sallen-Key lowpass filter circuit

Example Design

- Butterworth 2nd Order LPF

Special Case 1
(K = 1, Solve for C's)

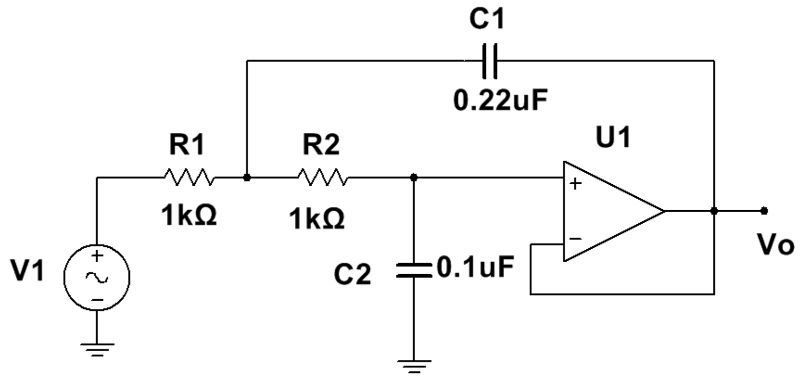
$$K = 1 \quad (R_3 = \infty, R_4 = 0)$$

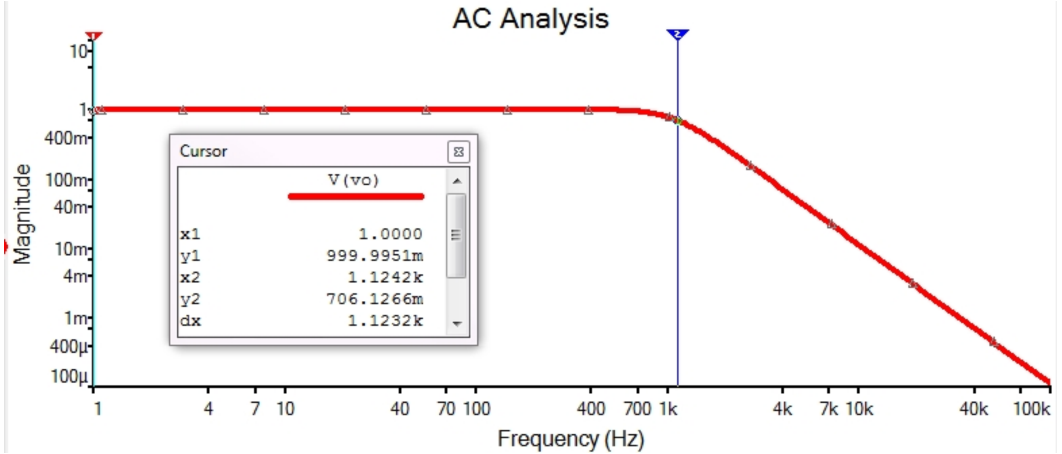
$$C_1 = \frac{Q}{\omega_o}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$C_2 = \frac{1}{Q\omega_o\left(R_1 + R_2\right)}$$

Can simplify with $R_1 = R_2$

Example Design





Summary

- Designed a second-order lowpass filter

Next Lesson

- Filter Demonstration

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Filtering Demonstration

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Demonstrate filtering of signals

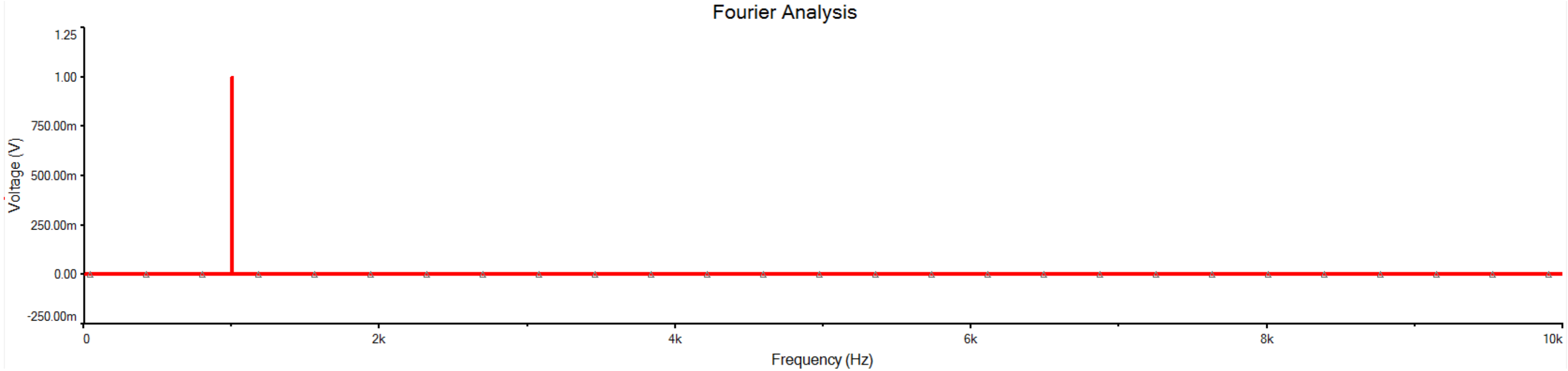
Previous Lesson

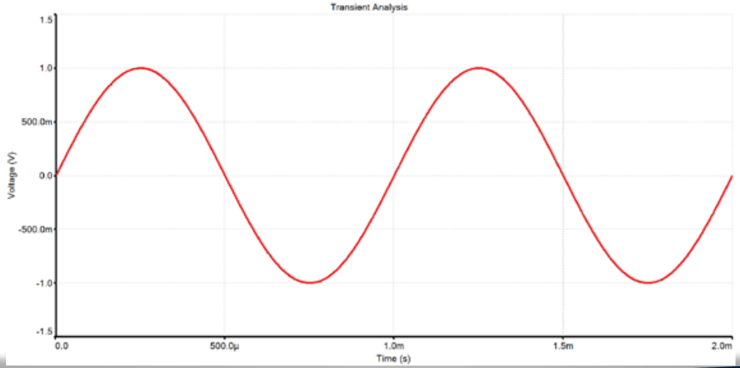
- Introduced second-order filter circuits

Lesson Objectives

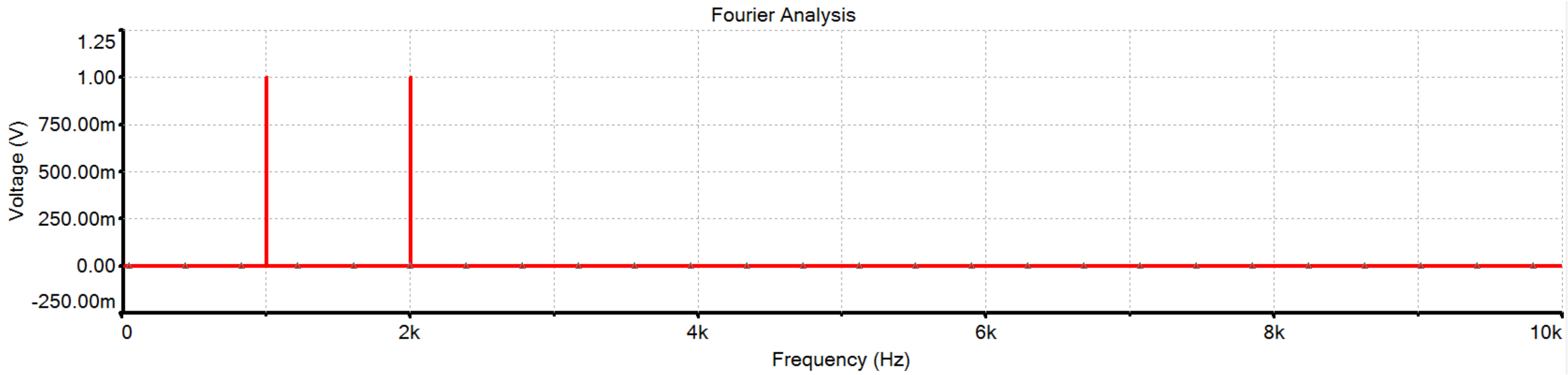
- Examine frequency spectra of signals
- Demonstrate filtering by a second-order filter circuit

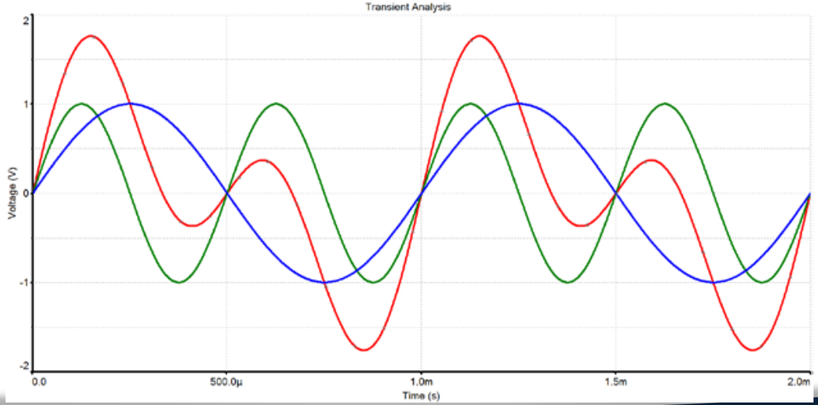
Spectrum of Sine Wave



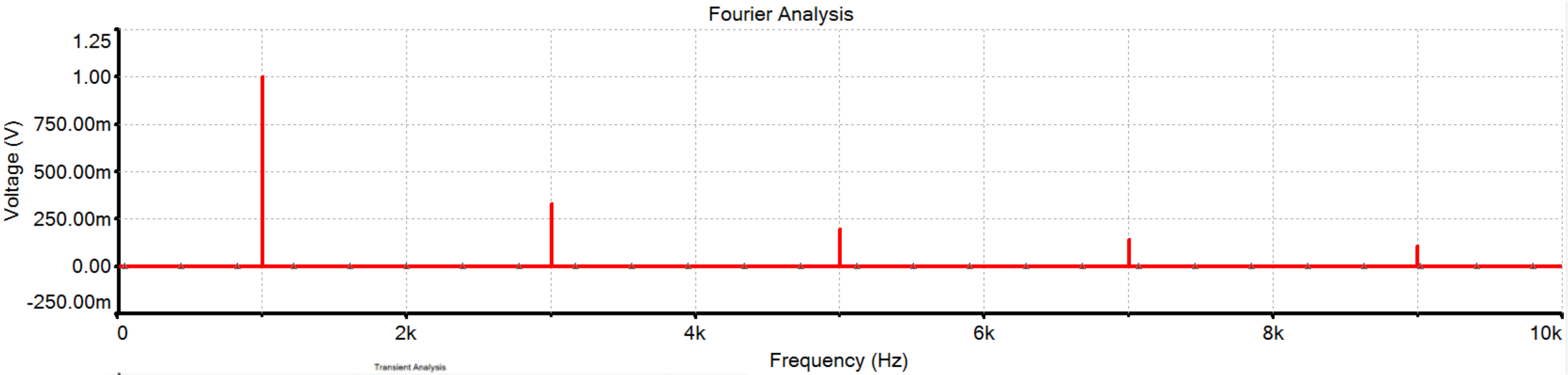


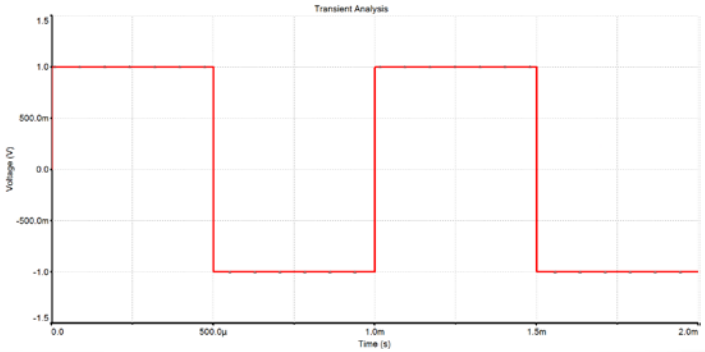
Spectrum of Sum of Two Sine Waves



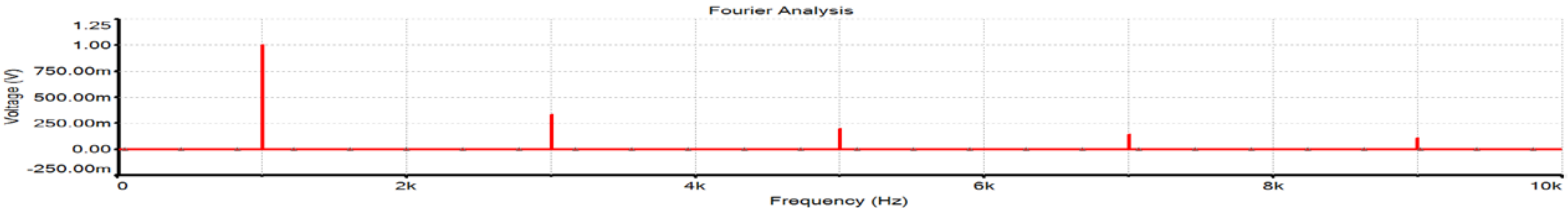


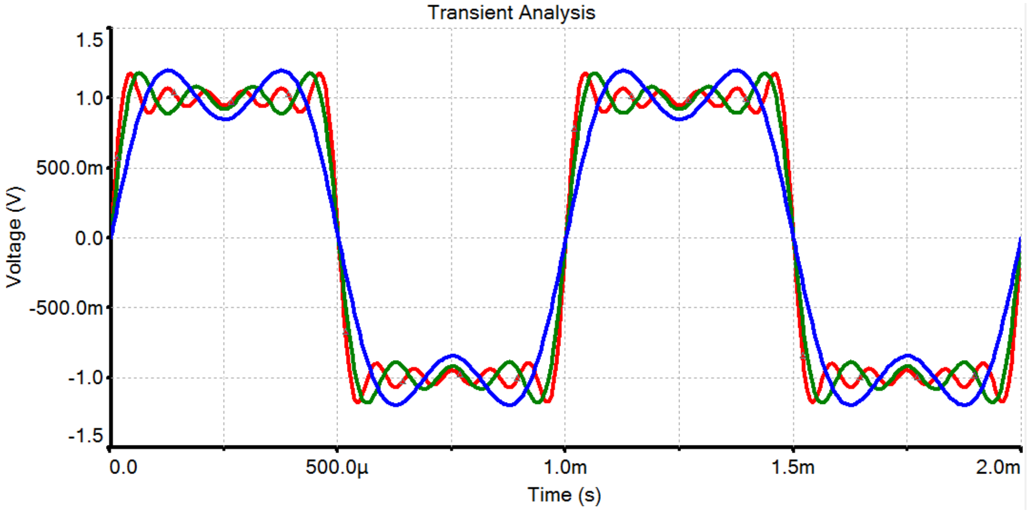
Spectrum of Square Wave



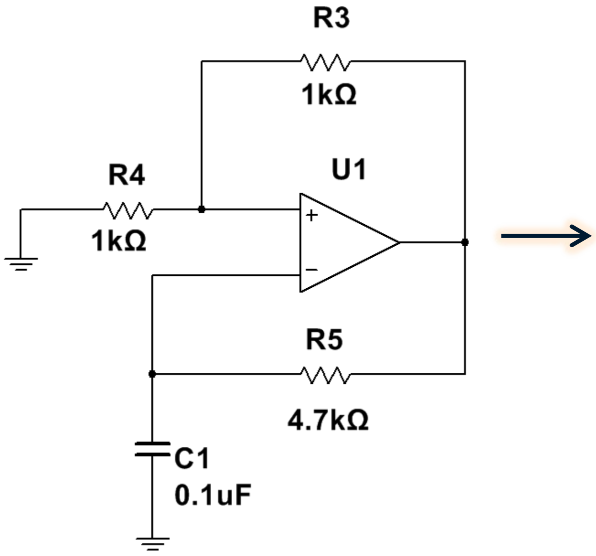


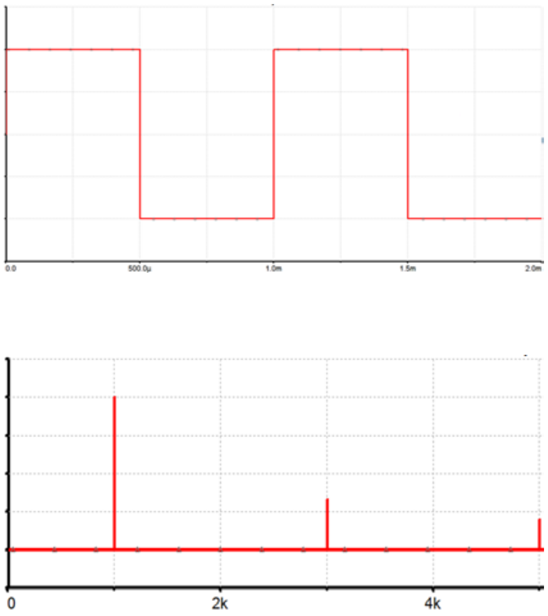
Spectrum of Square Wave



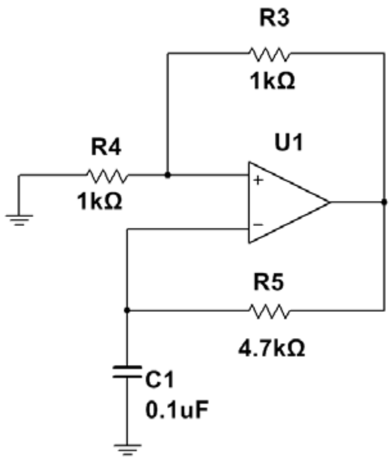


Relaxation Oscillator

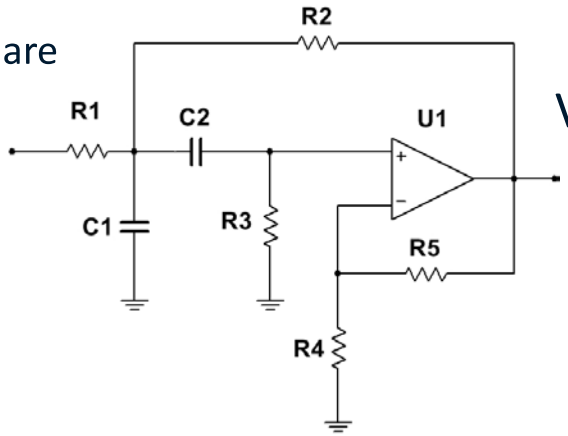




Measurements



1kHz Square Wave



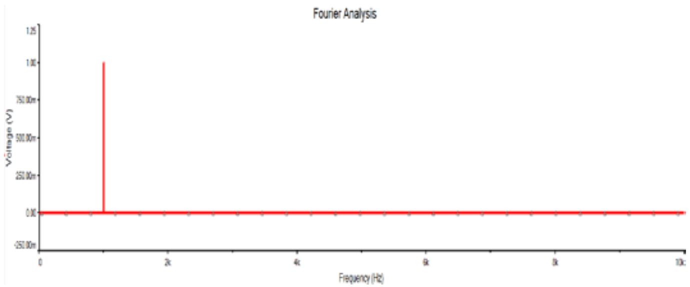
V_o

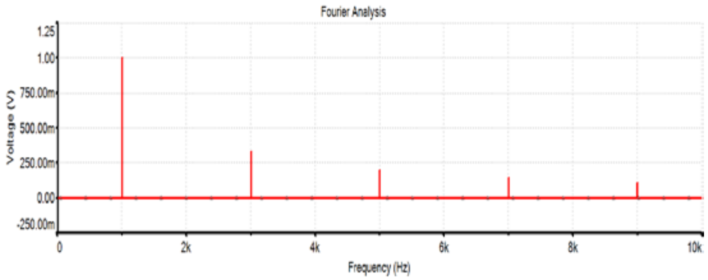
Relaxation Oscillator

f_0 = 1kHz Q=5 Sallen-Key BPF

Total Harmonic Distortion (THD)

$$THD = \frac{\sqrt{v_2^2 + v_3^2 + v_4^2 + \dots}}{v_1} * 100\%$$





Summary

- Introduced frequency spectra
- Examined physical circuit filtering performance